Lesson4: Operational space trajectory

A possible solution is similar to the methods use in the joint space.
We define points $x_e(t_k)$

$$X_e(t) = \begin{bmatrix} x_{e,1} \\ \vdots \\ x_{e,r} \end{bmatrix}$$

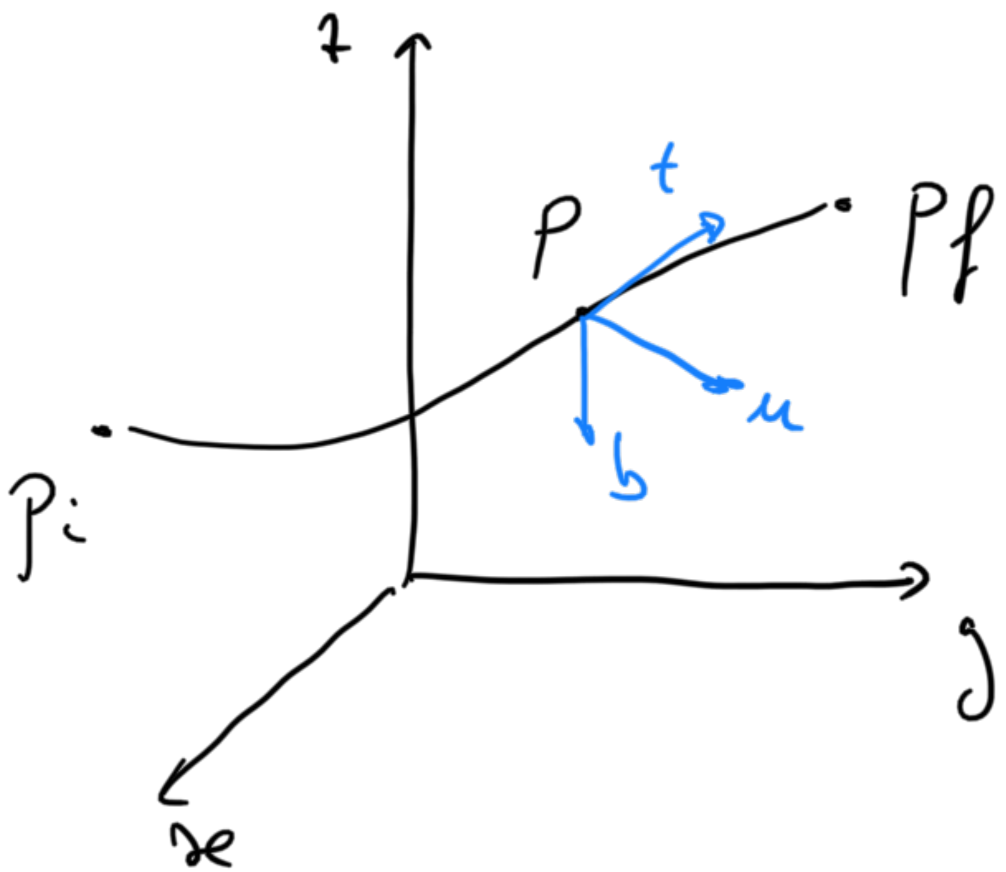
r dimention of the operational space

We then define sequence of cubic polynomials using e.g. SPLINES
However it might be important to define

an analytic expression f = the trajectory.

SOLUTION : DEFINE PATH PRIMITIVES

PATH PRIMITIVES



$p \in \mathbb{R}^{3\times1}$

$f(\sigma) : \sigma \in [\sigma_i, \sigma_f]$

$f(\sigma) \in C$

$$p = f(\sigma)$$

The sequence of values of p with σ

varying in $[\sigma_i, \sigma_f]$ is termed PATH in space.

$P = f(\sigma)$ is the PARAMETRIC REPRESENTATION of the path Γ and the scalar σ is the ~~paa~~ parameter. When σ increases, P move along Γ in a given direction.

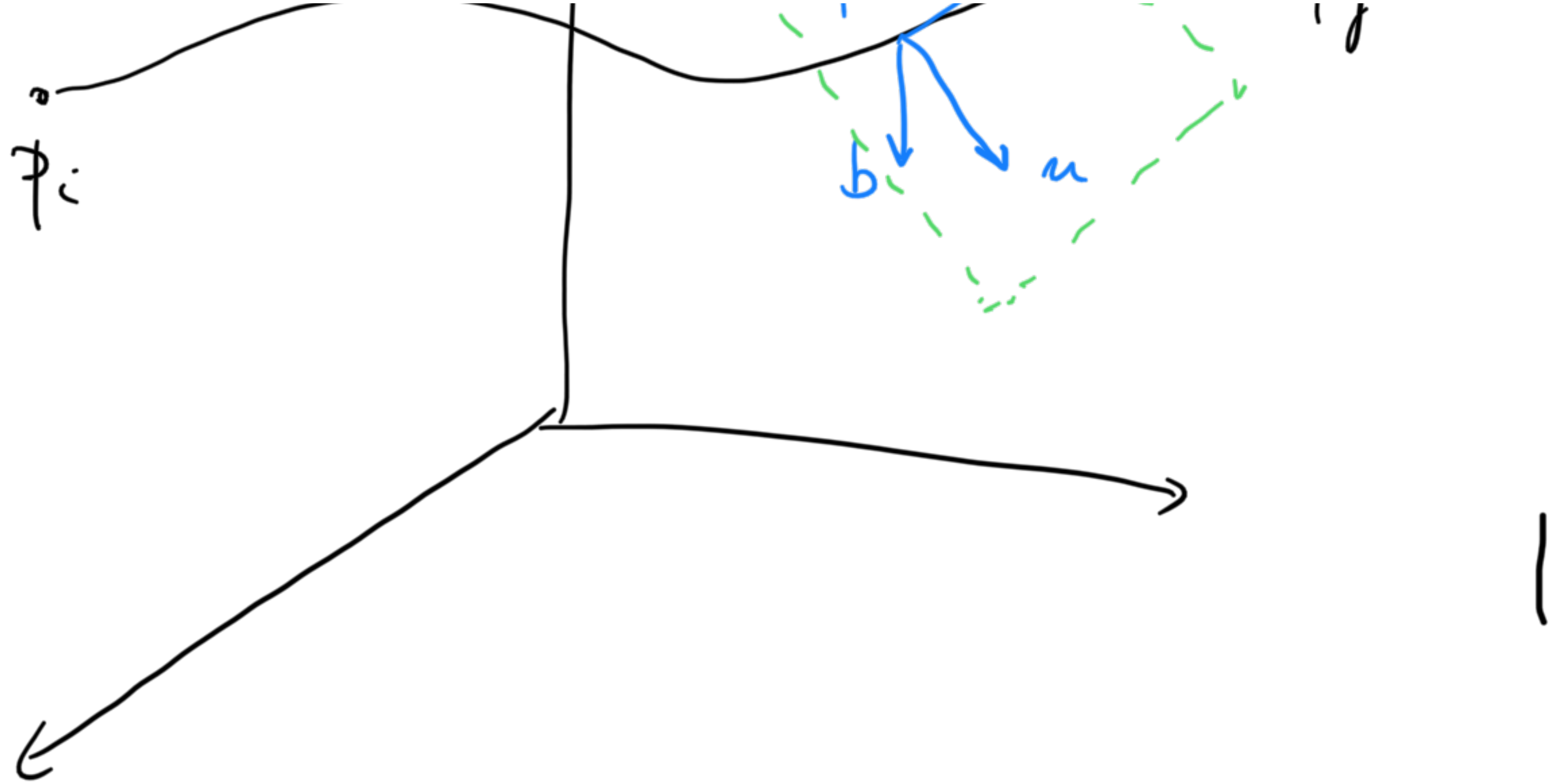
CLOSED PATH $\Rightarrow$ $P(\sigma_f) = P(\sigma_i)$

Let P_i be a point in Γ, with a fixed direction.

The ARC LENGTH S of the generic point P, is the length of the arc Γ with extremes P and P_i of P follows P_i or the opposite is P precedes P_i

P_i is the origin $\Rightarrow S = 0$

So that $P = f(s)$ can be a differentiable parametric representation of Γ



p allows the definition of a reference frame characterising the path.

t : tangent to the path in p

n : normal so to stay in the OSCULATING PLANE

b : binormal so to form a right-hand

... p ... a fu-nonal frame.

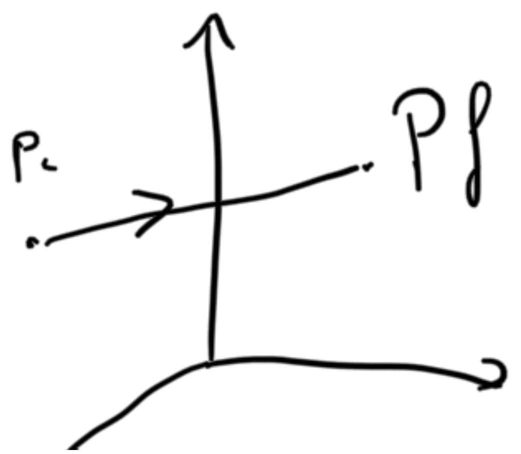
We can compute from Γ

$$t = \frac{dp}{ds}$$

$$n = \frac{1}{\left\| \frac{d^2 p}{d^2} \right\|} \frac{d^2 p}{ds^2}$$

$$b = t \times s$$

RECTILINEAR PATH



$$p(s) = p_i + \frac{s}{\|p_f - p_i\|}(p_f - p_i)$$

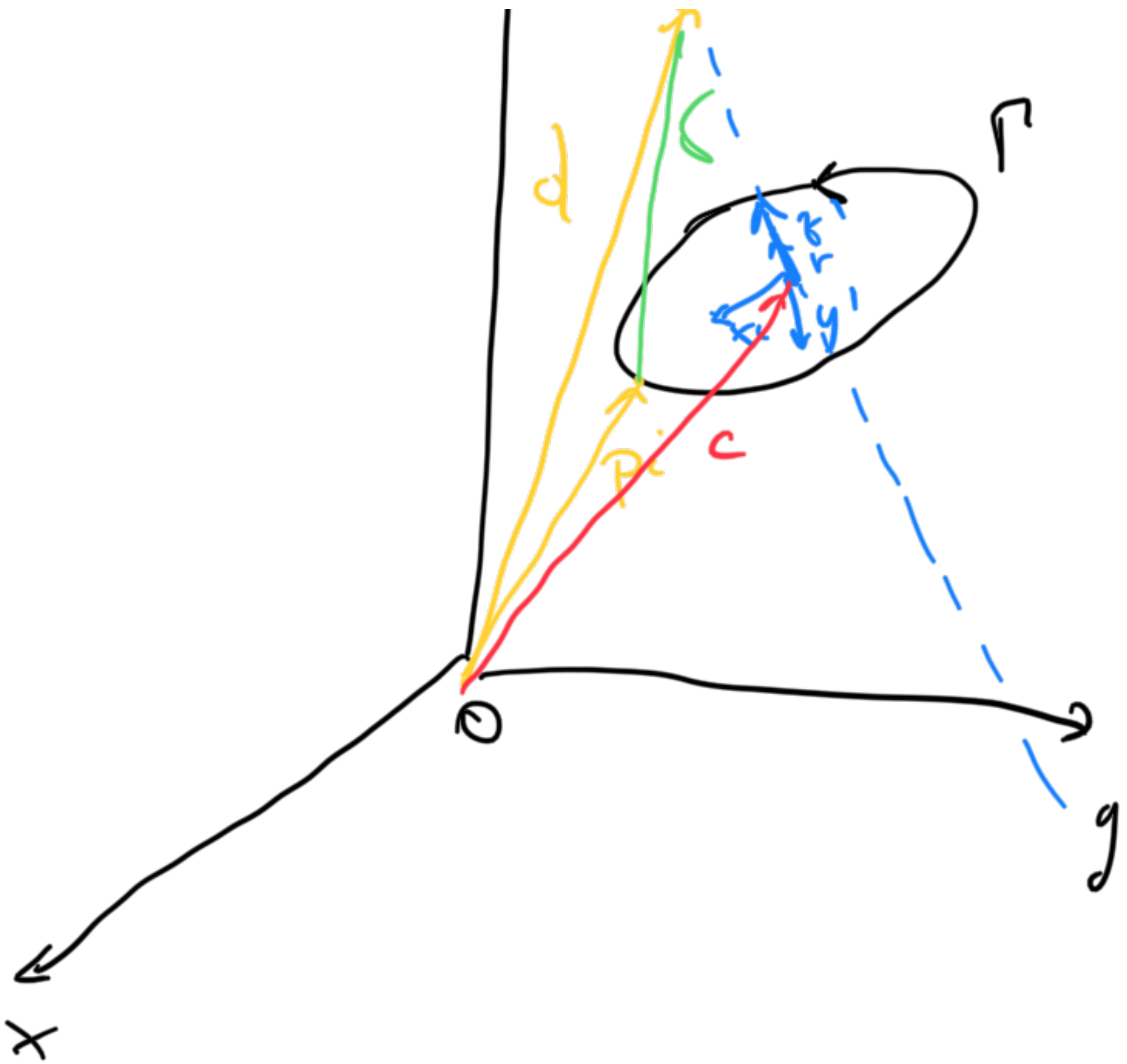
$$p(0) = p_i \qquad p(\|p_f - p_i\|) = p_f$$

$$\frac{dp}{ds} = \frac{1}{\|p_f - p_i\|}(p_f - p_i)$$

$$\frac{d^2p}{d^2s} = 0 \implies \text{It is not possible to uniquely define } (t, n, b)$$

CIRCULAR PATH

r, unit vector of the



axis in the circle.

d_i position vector of a point along the circle axis

p_i, position vector of a point on the circle.

c, position vector of the centre.

Let $r = p_i - d$. For p_i not to be on the axis (point), it must be

$$|\zeta^T r| < \|r\|$$

in this case

$$c = d + (\delta^T r) r$$

Let consider $O'(x', y', z')$ where O' coincide with the center of the circle. x' is oriented along $p_i - c$, z' is oriented along ~~r~~ and y' completes the right-handed frame.

The parametric representation of the circle is

$$p'(s) = \begin{bmatrix} \rho \cos(s/\rho) \\ \rho \sin(s/\rho) \\ 0 \end{bmatrix}$$

where $\rho = \|p_i - c\|$ is the radius of the circle and p_i the origin of the arc length.

If we report the representation w.r.t the original reference frame O (x, y, z), the path becomes

$$p(s) = C + R\, p'(s)$$

where C is expressed in O and R is the rotation matrix of O' w.r.t. O

$$\frac{dp}{ds} = R \begin{bmatrix} -\sin(s/\rho) \\ \cos(s/\rho) \\ 0 \end{bmatrix}$$

$$\frac{d^2 p}{ds^2} = R \begin{bmatrix} -\cos(s/\rho)/\rho \\ -\sin(s/\rho)/\rho \\ 0 \end{bmatrix}$$

POSITION

Generating a TRAJECTORY in the OPERATIONAL (TASK) SPACE means to determine a function $x_e(t)$ taking the END-EFFECTOR frame from the initial to the final pose in a time t_f

Let

Let $p_e = f(s) \in \mathbb{R}^{...}$ be the parametric representation of Γ. The origin of the E-E moves from p_i to p_f in t_f

The arc length goes from $s = 0$ @ $t = 0$ to $s = p_f$ @ $t = t_f$. $s(t)$ defines the timing law.

$s(t)$ can be compute using the techniques used in the joint space, e.g. cubic polynomials.

The velocity @ point p_e is the time derivative of p_e

$$\dot{p}_e = \dot{s}\,\frac{dp}{ds} = \dot{s}\,t$$

tangent vector

magnitude of the velocity

Velocity along a segment.

$$\dot{p}_e = \frac{\dot{s}}{\|p_f - p_i\|}(p_f - p_i) = \dot{s}\,t$$

$$\ddot{p}_e = \frac{\ddot{s}}{\|p_f - p_i\|}(p_f - p_i) = \ddot{s}\,t$$

Circle

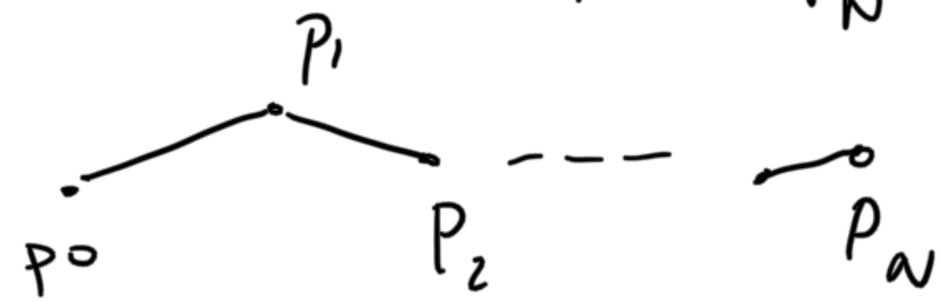
... in space

$$\dot{p}_e = R\begin{bmatrix} -\dot{s}\sin(s/\rho) \\ \dot{s}\cos(s/\rho) \\ 0 \end{bmatrix}$$

$$\ddot{p}_e = R\begin{bmatrix} -\dot{s}^2\cos(s/\rho)/\rho - \ddot{s}\sin(s/\rho) \\ -\dot{s}^2\sin(s/\rho)/\rho + \ddot{s}\cos(s/\rho) \\ 0 \end{bmatrix}$$

Both <u>tangential</u> and <u>centripetal</u> contributions

Consider a sequence of $N+1$ points $p_0, \dots, p_N$ connected by N segment



$$p_e = p_0 + \sum_{j=1}^{N} \frac{s_j}{\|p_j - p_{j-1}\|}(p_j - p_{j-1})$$

$$j = 1 \ldots N$$

s_j is the arc length associated with j-th segment of the path

$$s_j(t) = \begin{cases} 0 & 0 \le t \le t_{j-1} \\ s'(t) & t_{j-1} < t < t_j \\ \|p_j - p_{j-1}\| & t_j < t < t_f \end{cases}$$

with $t_0 = 0$ and $t_N = t_f$

\ can be an analytic function of cubic polynomials

VELOCITY $\dot{p}_e = \sum_{j=1}^{N} \frac{\dot{s}_j}{\|p_j - p_{j-1}\|}(p_j - p_{j-1}) = \sum_{j=1}^{N} \dot{s}_j t_j$

$\ddot{p}_e = \sum_{j=1}^{N} \frac{\ddot{s}_j}{\|p_j - p_{j-1}\|}(p_j - p_{j-1}) = \sum_{j=1}^{N} \ddot{s}_j t_j$

tangent unit vector of the j-th segment.

ORIENTATION

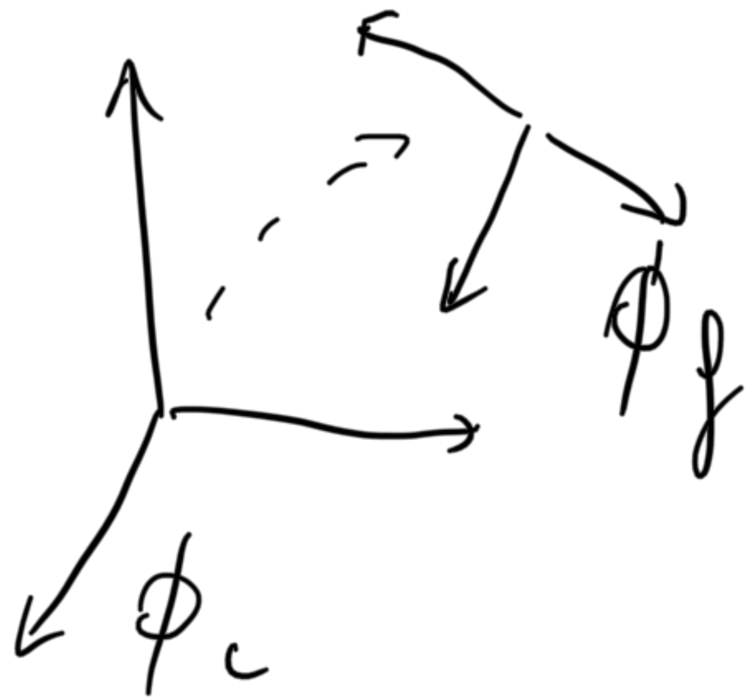
A linear interpolation of the unit vectors

n_e, s_e, a_e does not guarantee ORTHONORMALITY

We then need a representation of the rotations

EULER ANGLES

$$\phi_e = (\varphi, \vartheta, \psi)$$



$$\phi_e = \phi_i + \frac{s}{\|\phi_f - \phi_i\|}(\phi_f - \phi_i)$$

$$\phi_e = \frac{s}{\|\phi_f - \phi_i\|}(\phi_f - \phi_i)$$

$$\ddot{\phi}_e = \frac{\ddot{s}}{\|\phi_f - \phi_i\|}(\phi_f - \phi_i)$$

with $s(t)$ specified with the known techniques

It is also possible to use the AXIS-ANGLE REPRESENTATION → see book!